Problem 1. *Perceptron*

(i) We are given the following perceptron with weight vector $(\beta_0, \beta_1, \beta_2)^t = (2, 1, 1)^t$. Plot the partition in $\mathbb{R}^2$ that this perceptron creates, and indicate which region classify points as $+1$.

(ii) Amongst the following, which perceptrons return the same hyperplane and the same classification as the previous perceptron in *(i)*?

(a) $(1, 1/2, 1/2)^t$
(b) $(200, 100, 100)^t$
(c) $(\sqrt{2}, \sqrt{1}, \sqrt{1})^t$
(d) $(-2, -1, -1)^t$

Problem 2. *Perceptron learning*

(i) Apply the perceptron learning algorithm (with learning rate $\lambda = 1$) for the following training data, until convergence. Start with the initial vector $(\beta_0, \beta_1, \beta_2, \beta_3)^t = (1, 0, 0, 0)^t$. Apply the algorithm by visiting each point one after the other, in the order given below. For each step of the algorithm, write down the classification result and the updated weight vector.

$$\begin{aligned} x_1 &= (4, 3, 6)^t, \quad y_1 = -1 \\ x_2 &= (2, -2, 3)^t, \quad y_2 = 1 \\ x_3 &= (1, 0, -3)^t, \quad y_3 = 1 \\ x_4 &= (4, 2, 3)^t, \quad y_4 = -1 \end{aligned}$$

(ii) Apply the perceptron algorithm for the following training data, and show that cycles develop. Start with $(\beta_0, \beta_1, \beta_2)^t = (1, 0, 0)^t$. Conclude that this data cannot be solved with a single perceptron.

$$\begin{aligned} x_1 &= (1, 1)^t, \quad y_1 = 1 \\ x_2 &= (1, 0)^t, \quad y_2 = -1 \\ x_3 &= (0, 0)^t, \quad y_3 = 1 \\ x_4 &= (0, 1)^t, \quad y_4 = -1 \end{aligned}$$